Motion in 2 dimension

(additional cases for competitive exams only)

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Note : The notes given in this file is no substitute to the much detailed discussion held in the online/contact classes with active participation of students. It , at best, serves the purpose of ready reference for important concepts/derivations covered in the classes.

Two simultaneous projectiles

Consider two bodies (A and B) projected simultaneously from the same point. Let u_A and u_B be their initial velocities and θ_A and θ_B be their angles of projection of A and B respectively.

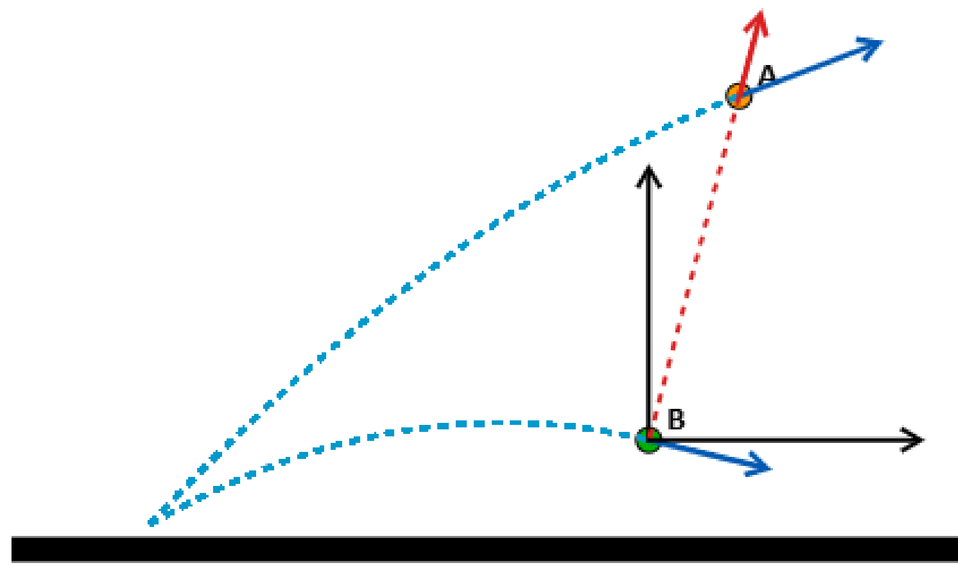
Using $\boldsymbol{S} = \boldsymbol{u}t + \frac{1}{2}\boldsymbol{a}t^2$ for x and y components of motion, the position of A and B are give by the relations

$$x_A = u_A \cos(\theta_A) t \qquad \text{i}$$

$$y_A = u_A \sin(\theta_A) t - \frac{1}{2} g t^2 \qquad \text{ii}$$

$$x_B = u_A \cos(\theta_B) t \qquad \text{iii}$$

$$y_B = u_B \sin(\theta_B) t - \frac{1}{2} g t^2 \qquad \text{iv}$$



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Two simultaneous projectiles

Horizontal and vertical displacements of A w.r.t. B are given by

$$x_{AB} = \left[u_A \cos(\theta_A) - u_B \cos(\theta_B)\right]t \tag{v}$$

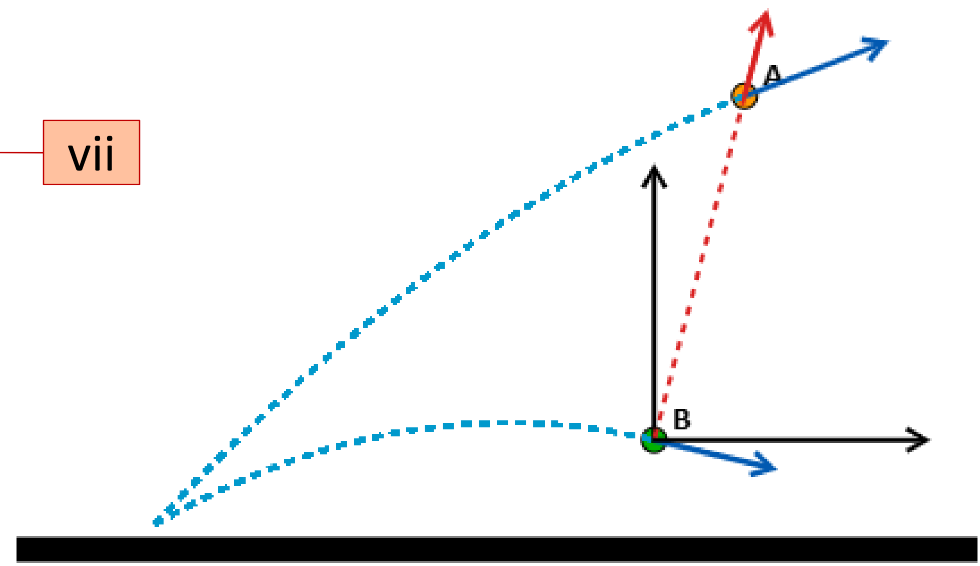
$$y_{AB} = \left[u_A \sin(\theta_A) - u_B \sin(\theta_B)\right]t \tag{vi}$$

Both relative horizontal and vertical displacements of B w.r.t. Increase linearly with time. Eliminating t from the above two equations the equation of path of B w.r.t. A is given by

$$y_{AB} = \left(\frac{u_A \sin(\theta_A) - u_B \sin(\theta_B)}{u_A \cos(\theta_A) - u_B \cos(\theta_B)}\right)x \tag{vii}$$

$$y_{AB} = m\ x_{AB}$$

The above equation is of the form of a straight line. Therefore motion of B w.r.t. A appears to be along a straight line.



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Trolley and projectile – case I

(Projection from ground)

Consider a trolley moving on a horizontal ground, with a uniform velocity v_T. Let a projectile be fired from the trolley at angle θ and initial velocity u w.r.t. the ground.

For observer on the ground

$$u_x = u\cos(\theta)$$

$$u_v = u\sin(\theta)$$

$$x = u\cos(\theta)\,t$$

$$y = u\sin(\theta)\,t - \frac{1}{2}gt^2$$

$$TOF = \frac{2u\sin(\theta)}{g}$$

$$R_t = u\cos(\theta) \times TOF$$

$$R_t = \frac{u^2\sin(2\theta)}{g} \qquad \text{i}$$

$$H = \frac{u^2\sin^2(\theta)}{2g} \qquad \text{ii}$$

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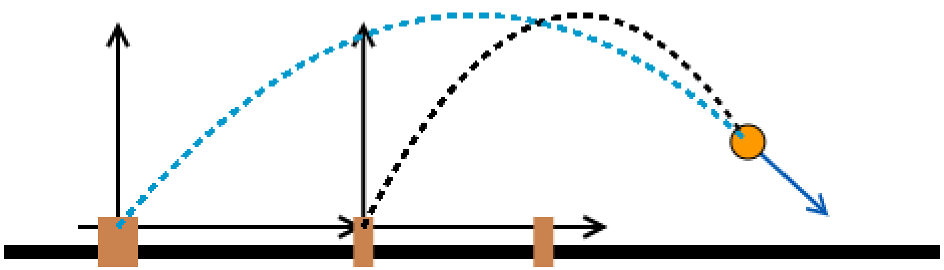

Trolley and projectile – case I

Click here for simulation

(Projection from ground)

Consider a trolley moving on a horizontal ground, with a uniform velocity v_T. Let a projectile be fired from the trolley at angle θ and initial velocity u w.r.t. the ground.

For observer on the trolley

$$\boldsymbol{v}_{\text{moving frame}} = \boldsymbol{v}_{\text{G}} - \boldsymbol{v}_{\text{frame of ref}}$$

$$u_x = u\cos(\theta) - v_t$$

$$u_y = u\sin(\theta)$$

$$x = [u\cos(\theta) - v_t]\,t$$

$$y = u\sin(\theta)\,t - \frac{1}{2}gt^2$$

$$TOF = \frac{2u\sin(\theta)}{g}$$

$$R_G = [u\cos(\theta) - v_t] \times TOF$$

$$R_G = [u\cos(\theta) - v_t]\frac{2u\sin(\theta)}{g} \qquad \text{iii}$$

$$H = \frac{u^2\sin^2(\theta)}{2g} \qquad \text{iv}$$

Range appears to be less when seen from trolley as the relative velocity of projectile w.r.t. the trolley is less. TOF & H are unaffected as there is no change in u_y.

Trolley and projectile – case II

Click here for simulation

(Projection from trolley)

Consider a trolley moving on a horizontal ground, with a uniform velocity v_T. Let a projectile be fired from the trolley at angle θ and initial velocity u w.r.t. the trolley.

For observer on the ground

$$\boldsymbol{v}_G = \boldsymbol{v}_{\text{moving frame}} + \boldsymbol{v}_{\text{frame of ref}}$$

$$u_x = u\cos(\theta) + v_t$$

$$u_y = u\sin(\theta)$$

$$x = \left[u\cos(\theta) + v_t\right] t$$

$$y = u\sin(\theta)\, t - \frac{1}{2} g t^2$$

$$TOF = \frac{2u\sin(\theta)}{g}$$

$$R_G = \left[u\cos(\theta) + v_t\right] \times TOF$$

$$R_G = \left[u\cos(\theta) + v_t\right] \frac{2u\sin(\theta)}{g} \qquad \text{i}$$

$$H = \frac{u^2\sin^2(\theta)}{2g} \qquad \text{ii}$$

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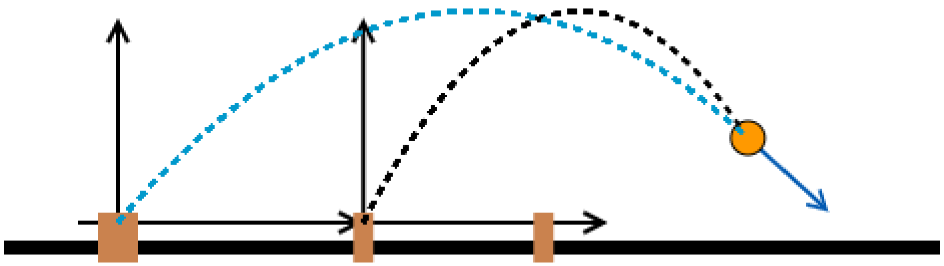


Trolley and projectile – case II

Click here for simulation

(Projection from trolley)

Consider a trolley moving on a horizontal ground, with a uniform velocity v_T. Let a projectile be fired from the trolley at angle θ and initial velocity u w.r.t. the trolley.

For observer on trolley

$$u_x = u\cos(\theta)$$

$$u_y = u\sin(\theta)$$

$$x = u\cos(\theta)\,t$$

$$y = u\sin(\theta)\,t - \frac{1}{2}gt^2$$

$$TOF = \frac{2u\sin(\theta)}{g}$$

$$R_t = u\cos(\theta)\times TOF$$

$$R_t = \frac{u^2\sin(2\theta)}{g}$$

iii

$$H = \frac{u^2\sin^2(\theta)}{2g}$$

iv

Range appears to be more when seen from ground as velocity of projectile w.r.t. Is more . TOF & H are unaffected as there is no change in u_y.

Projection on an inclined plane

Consider a projectile projected with initial velocity u at angle θ w.r.t. the ground towards an inclined plane. Let α be the angle of inclination of the plane w.r.t. the ground. Problem is greatly simplified if we resolve u & g into components parallel and perpendicular to the plane, i.e.

$$u_{||} = u\cos(\theta - \alpha) \qquad u_{\perp} = u\sin(\theta - \alpha)$$

$$g_{||} = g\sin(\alpha) \qquad g_{\perp} = g\cos(\alpha)$$

TOF

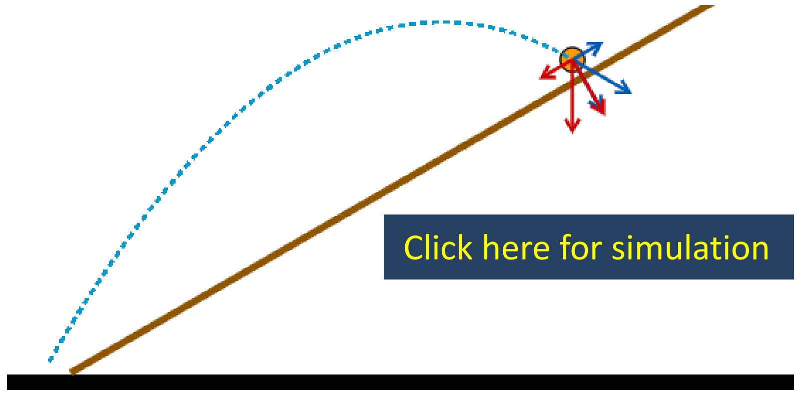
It is the time at which the perpendicular distance w.r.t the plane i.e. $S_{\perp}$ becomes zero

$$S_{\perp} = u_{\perp} t + \frac{1}{2} g_{\perp} t^2$$

$$0 = u\sin(\theta - \alpha)t + \frac{1}{2} g\cos(\alpha)t^2$$

$$TOF = \frac{2u\sin(\theta - \alpha)}{g\cos(\alpha)}$$

i



Projection on an inclined plane

<u>Range along the plane</u>

$$R_{||} = u\cos(\theta-\alpha)\times TOF$$

$$R_{||} = u\cos(\theta-\alpha)\times \frac{2u\sin(\theta-\alpha)}{g\cos(\alpha)}$$

$$R_{||} = \frac{u^2\sin 2(\theta-\alpha)}{g\cos(\alpha)}$$ ii

<u>TOA w.r.t the plane</u>

Using the condition that $v_\perp$ = 0 at the highest point

$$v_\perp = u\sin(\theta-\alpha) - g\cos(\alpha)t$$

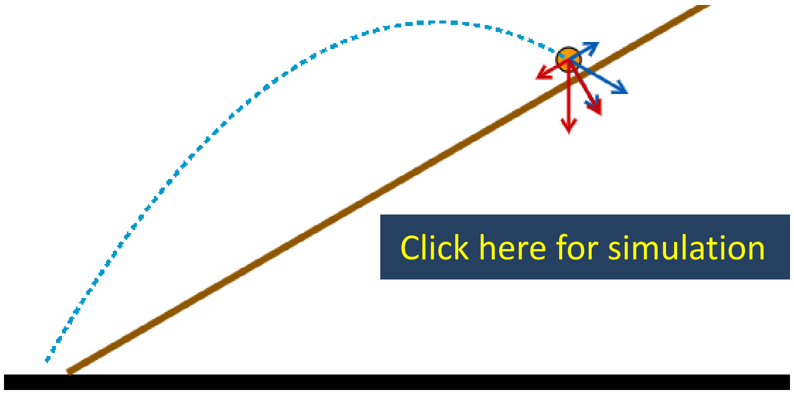
$$t = \frac{u\sin(\theta-\alpha)}{g\cos(\alpha)}$$ iii

<u>$H_\perp$ w.r.t the plane</u>

Using $\boldsymbol{S} = \boldsymbol{u}t + \frac{1}{2}\boldsymbol{a}t^2$ and TOA for ⊥ component of motion

$$H_\perp = u_\perp \times TOA - \frac{1}{2}g_\perp (TOA)^2$$

$$H_\perp = \frac{u^2\sin^2(\theta-\alpha)}{2g\cos(\alpha)}$$ iv



The boat – river problem

Consider a river of width W in which water is flowing with uniform velocity v_R. Consider a boat which can move with a velocity v_B (i.e. " in still waters ") and at an angle θ w.r.t the flow of water. The boat attempts to cross the river.

Velocity of boat w.r.t. the ground

$$\boldsymbol{v}_G = \boldsymbol{v}_{\text{moving frame}} + \boldsymbol{v}_{\text{frame of ref}}$$

$$\boldsymbol{v}_{BG} = \boldsymbol{v}_B + \boldsymbol{v}_W$$

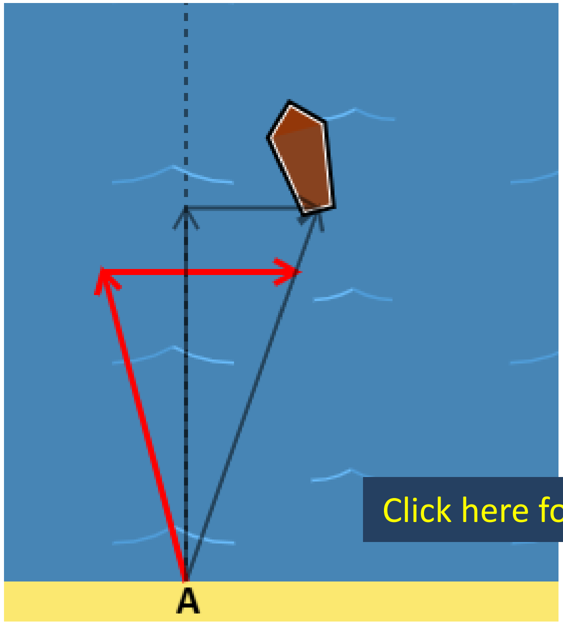
$$v_{BG} = \sqrt{v_B^{\,2} + v_W^{\,2} + 2 v_B v_W \cos(\theta)}$$

Angle of displacement w.r.t the normal to the bank

$$\alpha = \tan^{-1}\left(\frac{v_B \sin(\theta)}{v_W + v_B \cos(\theta)}\right)$$

Time taken to cross the river

$$t = \frac{W}{v_B \sin(\theta)}$$



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